The Mechanical Viscosity of Fluids.

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(From the National Physical Laboratory.)

The experiments described in the following paper were undertaken in order to investigate the relation between shearing stress and rate of distortion in fluids which are in eddying or sinuous motion, that is, motion in which the frictional resistance, at the boundaries of the solid over which they move, varies approximately as the square of the relative velocity, as distinguished from that steady or laminar motion in which the frictional resistance is proportional to the first power of the relative speed.

This shearing stress has been called by Osborne Reynolds* "mechanical viscosity," *i.e.*, a "viscosity arising from the molar motion of the fluid and which is not a property of the fluid independent of its motion as is its physical viscosity." Thus, to quote Reynolds' statement, in the eddying motion of a fluid in a parallel channel, "although the mean motion at any point taken over a sufficient time is parallel to the axis of the pipe, it is made up of a succession of motions crossing the pipe in different directions." In this case, the shearing stress at this point on a cylindrical surface coaxial with the pipe "will include the momentum per second parallel to the pipe carried by the cross streams across the surface on which this shearing stress is measured." On the other hand, "the coefficient of physical viscosity is the coefficient of instantaneous resistance to distortion at a point moving with the fluid."

As far as the author is aware, no attempts have yet been made to measure the shearing stress in fluids which are in eddying motion, but Osborne Reynolds has assumed that this stress is proportional to the mean speed of flow, and is also a function of the dimensions of the channel in which the motion takes place.

For experimental purposes, it appeared obvious that the conditions under which the phenomenon could be most conveniently studied were those of the motion of air or water in parallel pipes of circular cross section, and as all the appliances used for the determination of the friction of air in pipes at the National Physical Laboratory were available, air was the fluid chosen for the experiments. Another reason for the choice of air was that in Threlfall's* observations on the distribution of mean axial velocity in air flowing through large pipes this distribution at any cross section appeared to be independent of the rate of flow. Further, although in Threlfall's experiments the general form of the velocity curve does not appear to have been investigated, yet the observed values of the radial gradient of velocity gave some hope of the possibility of an accurate determination of the rate of distortion. If this was found to be the case, a further determination of the variation of pressure along and across the pipe would furnish the data necessary for the estimation of the ratio of the shearing stress to the rate of distortion.

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* "Theory of Lubrication," 'Phil. Trans.,' 1886, p. 165.

Thus, if F is the average shearing stress on the surface of any cylindrical portion of radius r of the fluid coaxial with the pipe through which it flows, and v the mean axial velocity of the fluid at this radius; then, writing $F = \mu' \, dv/dr$, the object of the experiments was the determination of μ', which may be called the coefficient of mechanical viscosity, as a function of v and the dimensions of the pipe.

The Method of Establishing the Air Current and Estimating the Values of F *and* v.

The arrangement of one of the experimental pipes and the air fan used to set up the current is shown in fig. I. The air fan discharges into a horizontal pipe 3·5 metres in length. This pipe is connected by a bend to a vertical pipe 5·5 metres high. The experimental portion, 61 cm. long, is at the upper extremity of the vertical pipe. It was found by experiment that this length of outlet was sufficient to set up the limiting distribution of the velocity across the pipe as long as the diameter did not exceed 7·5 cm. The fan was driven by an electric motor, which was provided with a sensitive speed indicator, and during the observations at any given speed of flow the speed of the motor was kept constant by hand regulation of a carbon resistance in the armature circuit of the motor.

The values of v were determined from the difference between the pressure in a small Pitot tube facing the current and that in a small orifice in the side of the pipe. The Pitot tube was of rectangular section, the dimensions at the orifice being :—

In the direction of the radius of the pipe	External, 0·33 mm. Internal, 0·25 „
Perpendicular to the radius of the pipe.........	External, 1·27 mm. Internal, 1·20 „

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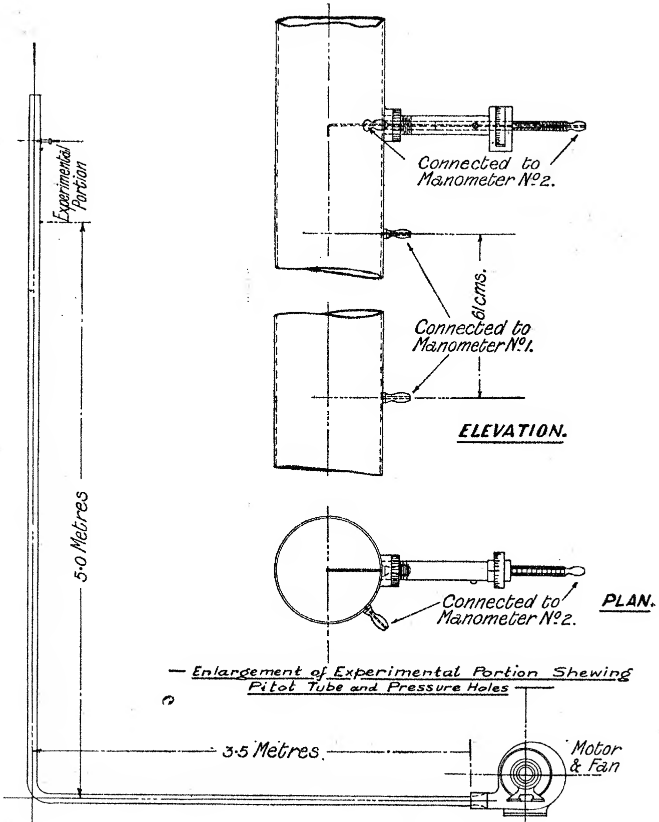
* "Motion of Gases in Pipes," 'Inst. Mech. Engineers Proc.,' 1904.

This tube was fixed to a micrometer, and could be moved across a diameter of the pipe. The pressure difference was measured by a sensitive oil and water tilting gauge, whose indications could be relied upon within an accuracy of 0·005 mm. of water. The final calibration of the Pitot tube was made by a comparison of its readings with the measured discharge of the pipe through a gas meter.

The determination of the shearing stress on any cylindrical portion of the fluid of radius r was made by a direct measurement of the drop of pressure at the surface along the pipe, together with the measurement of the variation of pressure along the radius. The former was observed by taking the pressure difference in two small tubes let into the sides of the pipe. As regards the latter, it has been shown by Osborne Reynolds* that although in steady motion $dp/dr = 0$, when the motion is sinuous $d/dr\,.\,(p+\rho w^2) = 0$, where w


Experimental Portion
Connected to Manometer No. 2.
61 cms.
Connected to Manometer No. 1.
ELEVATION.
5·0 Metres
Connected to Manometer No. 2.
PLAN.
3·5 Metres
Motor & Fan

— Enlargement of Experimental Portion Shewing Pitot Tube and Pressure Holes

— General Arrangement of Air Pipe and Fan.—

Fig. I.

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* 'Nature,' 1898, p. 468.

is the component of sinuous motion normal to the surface. To detect such a variation, a small tube of radius 0·4 mm., with a hole in its side connected to a sensitive gauge, was moved across a diameter, but no variation of pressure could be found up to a distance of 2·0 mm. from the walls. It was therefore assumed in the reductions that the pressure was constant across any section, and that the shearing stress was, therefore, proportional to the radius.

(A) EXPERIMENTS ON ARTIFICIALLY ROUGHENED PIPES.

As in ordinary smooth pipes it is known that the frictional resistance varies as a lower power of the velocity than the square, and that in consequence, as shown by Lord Rayleigh,* the friction is given by

$$F = \rho v_c^2 f(\nu / v_c l),$$

where ν = kinematical coefficient of viscosity, l = the linear dimension of the pipe, and v_c the velocity at the axis of the pipe.

In order to simplify the problem, an attempt was made to eliminate any effect due to a variation in $f(\nu / v_c l)$ by artificially roughening the pipes used until the friction was proportional to the square of the velocity. This was done by cutting a double screw-thread on the inside surface of the pipes, and so varying the pitch and depth that the surfaces of pipes of different diameters were geometrically similar.

After several trials, two pipes were produced, of diameters 7·35 and 5·08 cm., in which the friction varied as the square of the velocity, and consequently the friction per unit area at the same velocities was found to be the same for each pipe, the numerical value being $4 \cdot 6\ v_c^2 \times 10^{-6}$ dynes per square centimetre. The results of a large number of experiments on these two pipes were as follows:—

Effect of Velocity of Flow on the Radial Distribution of Velocity.

To test the constancy of the velocity distribution in a radial direction, two sets of observations were made at the highest and lowest centre filament velocities which could be satisfactorily obtained. These were 2220 and 546 cm. per second respectively. The two sets of reduced velocities for the 7·35 cm. diameter pipe are plotted in fig. II, and it will be seen that for this range of speed there is no definite change in the form of the curve, which, from the centre up to a distance of about 3 mm. from the wall of the pipe, is a parabola with its vertex in the axis of the pipe, the equation to which may be written

$$v = v_c - Ar^2.$$

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* 'Report of Advisory Committee for Aeronautics,' 1909–10, p. 38; also 'Phil. Mag.,' 1892, p. 59.

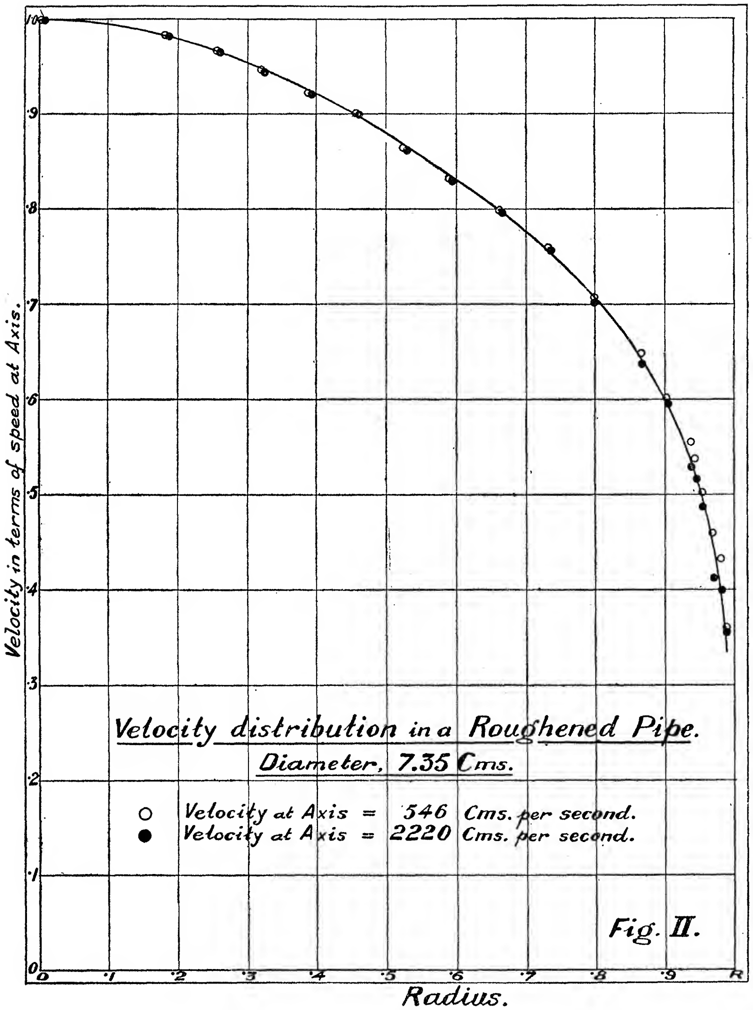

Fig. II.

From the constancy of the form of the curve, A is proportional to v_c, so that for any fixed value of the radius the value of dv/dr, which varies as A, is proportional to v_c. The plotted values of the velocities at points on the radius nearer than 3 mm. from the wall are found to be appreciably below the parabolic curve through the inner values, which apparently indicates that the screw-cutting of the surface has not entirely broken up the region of viscous flow at the boundary.

Effect of Radius of Pipe on the Distribution of Velocity.

As regards the effect of the diameter of the pipe on the velocity distribution, it was found that if in the two pipes the velocities at the axes were made proportional to the corresponding diameters, the two velocity curves thus determined were precisely similar, so that for corresponding points on the radii the values of dv/dr were the same for each pipe.

This result is shown in fig. III, in which the velocity curve for the 7·35 cm. pipe, with a velocity at the axis of 2210 cm. per second, is plotted to scale

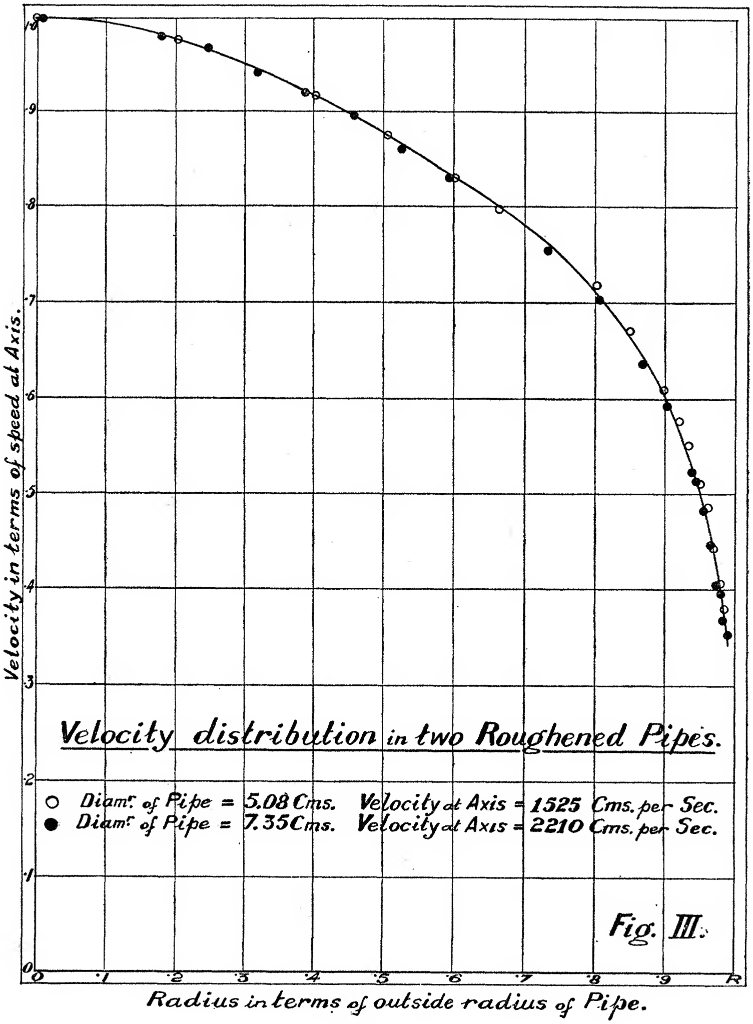

Fig. III.

and the corresponding curve for the 5·08 cm. pipe, with a velocity at the axis of 1525 cm. per second, is plotted with the abscissæ and ordinates strained in the ratio of the diameters, *i.e.*, 1·45. It is evident from the coincidence of the two curves that the condition of similarity is fulfilled. The actual values of the velocities are also tabulated in Table I.

Table I.—Reduced Values of the Velocities in the two Roughened Pipes (v_c/l constant).

Diameter 5 ·08 cm.		Diameter 7 ·35 cm.	
Radius.	Velocity in cm. per sec.	Radius.	Velocity in cm. per sec.
0	1526	0	2210
0 ·52	1495	0 ·661	2172
0 ·77	1450	0 ·917	2132
1 ·02	1404	1 ·17	2081
1 ·28	1342	1 ·42	2038
1 ·53	1278	1 ·68	1986
1 ·785	1200	1 ·92	1914
2 ·04	1092	2 ·18	1842
2 ·165	1023	2 ·44	1762
2 ·29	937	2 ·71	1674
2 ·345	887	2 ·95	1553
2 ·380	845	3 ·20	1400
2 ·42	785	3 ·33	1311
2 ·445	758	3 ·46	1162
2 ·470	688	3 ·485	1137
2 ·495	628	3 ·52	1074
2 ·51	585	3 ·56	1000
		3 ·58	946
		3 ·61	887
		3 ·635	811
		3 ·65	780

From the foregoing results the following conclusions for these roughened pipes are derived:—

(*a*) In such a pipe, through which air is moving at a speed above the critical value, it follows from the parabolic form of the curve that the mechanical viscosity, as defined above, is constant across the pipe* up to within a relatively small distance from the boundary.

(*b*) Considering the variations in the shearing force on any cylindrical surface coaxial with the pipe due to variations in the velocity of flow, it is evident that, since the value of dv/dr is proportional to v_c, the velocity of the centre filament, and the shearing force varies as the square of the velocity, therefore μ' is directly proportional to the velocity of flow.

(*c*) In two pipes in which the velocities of flow are made proportional to the diameters the values of dv/dr on two cylindrical surfaces, one in each pipe, whose linear dimensions are proportional to these diameters, are the same. It follows, therefore, that, since the respective shearing forces vary as the squares of the corresponding velocities of flow, the values of μ' must, for different pipes, be directly proportional to the diameters.

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* The equation to the velocity curve being $v = v_c - \mathrm{A}r^2$, $dv/dr = -2\mathrm{A}r$; and since F is known to be proportional to r, it follows that μ' in the relation $\mathrm{F} = \mu' \, dv/dr$ is constant.

It appears, therefore, that the expression for the coefficient of mechanical viscosity in the artificially roughened pipes considered may be written

$$\mu' = k\,v_c l,$$

where v_c is the velocity of flow at the axis of the pipe, l the linear dimension of the pipe, and k is a constant depending on the extent of the eddying motion, *i.e.*, the roughness of the surface.

(B) EXPERIMENTS ON ORDINARY SMOOTH PIPES.

For these experiments two smooth brass pipes of approximately the same diameters as the artificially roughened ones were used. (These were 4·93 and 7·4 cm. diameter.) As in the case of the rough pipes, two sets of observations,

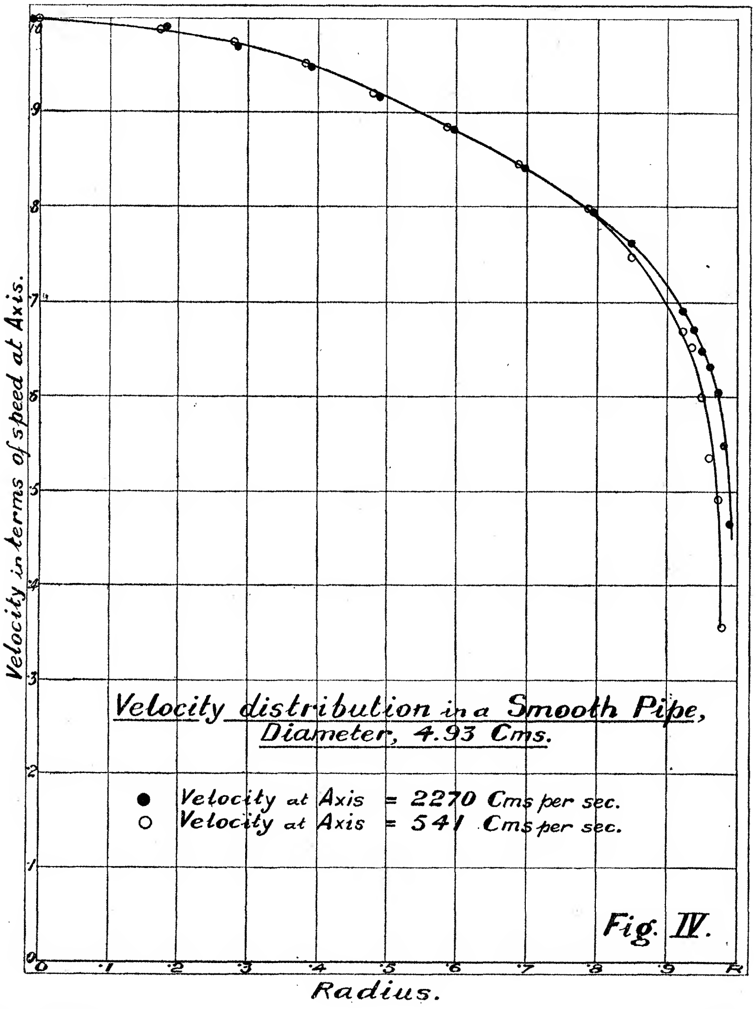
Velocity distribution in a Smooth Pipe, Diameter, 4.93 Cms.

Fig. IV.

one at the highest speed of flow obtainable and the other at the lowest, were made on one of the pipes, and the distributions of axial velocity in the two cases are shown in fig. IV. It will be seen that, for a region extending from the axis up to a value of the radius of approximately 0·8 R, the distributions are identical, but that beyond this radius the curves separate, indicating apparently a region of viscous flow near the walls, in which the slope of the velocity curves necessarily increases at a greater rate than the centre filament velocity, since the resistance varies as a power of the speed greater than unity. It may be mentioned that in the earlier experiments the corresponding curves plotted from the observations were not anywhere coincident, except at the axis, but, as a possible explanation of this was the insufficient length of the pipe between the fan and the portion under observation, further experiments were made with an increased length of pipe, and it was ultimately found that with a total length of 9 metres there was practical coincidence within the region mentioned. It is of course possible that even then the coincidence was not exact, but the difference, if any, was too small to be detected by the gauges. It was evident, therefore, that in the case of smooth pipes the region of viscous flow was of considerably greater area than in the case of the roughened pipes, in which it appeared to be nearly destroyed. It was, however, found that the central portion of the velocity distribution curve in the smooth pipes up to a radius of 0·8 R was parabolic in form, as in the case of the roughened pipes, thus indicating that the value of μ' was constant up to this limit.

As regards the effect of the size of the pipe, a precisely similar investigation to that used for the roughened pipes showed that, for two pipes in which the centre filament velocities were proportional to the respective diameters, the velocity distributions were strictly similar throughout the central portion of the flow referred to above.*

Considering, therefore, this region, since the values of dv/dr for any given value of the radius are proportional to the velocity of flow, and since the friction per unit area at this radius is known to be proportional to $\rho v_c^2 f(\nu/v_c l)$, it is evident that in the case of the smooth pipe μ' is proportional to $v_c f(\nu/v_c l)$. Further, since for two smooth pipes in which the centre filament velocities are proportional to the respective diameters the values of dv/dr at corresponding points in the central region are the same, it follows that μ' is also proportional to the external diameter of the pipe, and the complete expression for the mechanical viscosity for the region in question is

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* The two velocity curves for this case have not been plotted as in the case of the rough pipes (fig. III), because the difference in the radii is too small to bring out their divergence in the region of viscous flow. The values of the velocities determined are given in Table II.

$$\mu' = cv_c l f(v/v_c l).$$

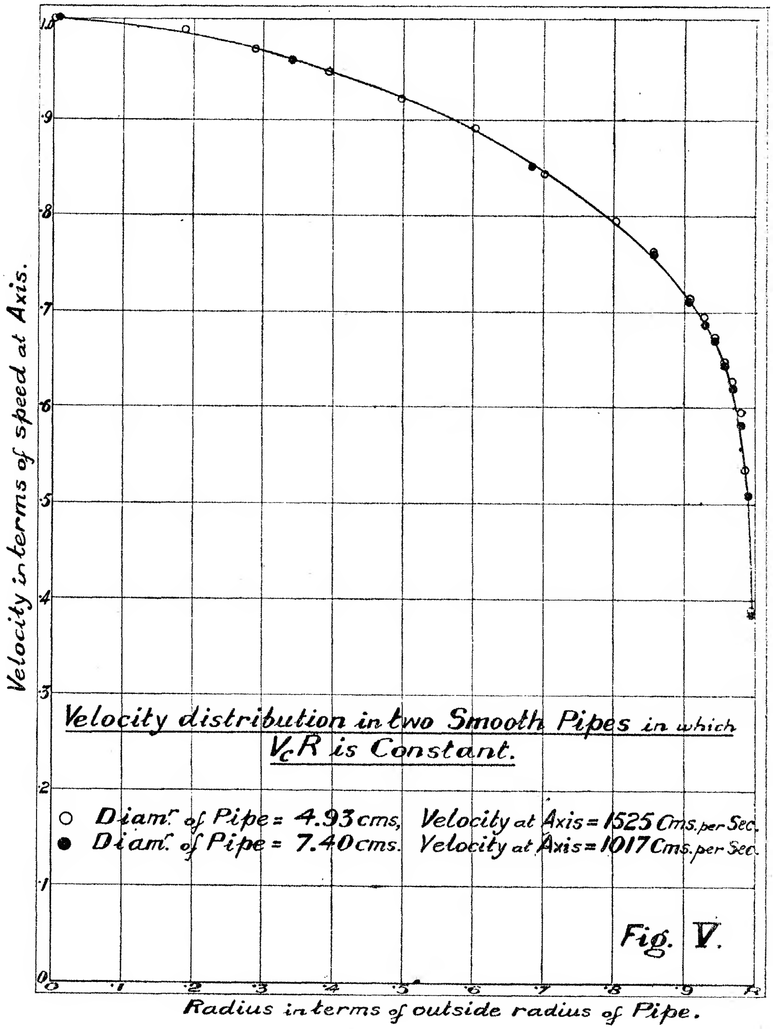
where c is a constant. This is, of course, in agreement with the value found for the roughened pipes, as in the latter case $f(v/v_c l)$ was a constant quantity for all values of v_c.

Table II.—Reduced Values of the Velocities in the two Smooth Pipes for Experiments in which $v_c l$ is constant (columns (1) and (2)) and v_c/l is constant (columns (1) and (3)).

Diameter 4·93 cm.		Diameter 7·4 cm.		
Radius.	Velocity in cm. per sec. (1)	Radius.	Velocity in cm. per sec. (2)	Velocity in cm. per sec. (3)
0	1525	0	1017	2215
0·45	1505	1·245	976	2120
0·70	1475	2·51	862	1892
0·96	1440	3·15	770	1693
1·21	1395	3·34	720	1605
1·47	1346	3·42	700	1556
1·72	1278	3·475	680	1510
1·97	1202	3·53	652	1453
2·10	1149	3·57	629	1407
2·23	1084	3·61	592	1344
2·28	1049	3·65	516	1248
2·32	1025	3·67	392	1098
2·35	984			
2·38	957			
2·41	908			
2·43	814			
2·44	592			

It appears, therefore, that the general character of the flow of air in pipes consists of a central region in which the ratio of the shearing stress to the rate of change of distortion is constant for any particular value of the flow, and that in the neighbourhood of the walls this alters rapidly until it becomes equal to the physical viscosity at the boundary.

In such a composite state of motion as this, it is clear that exact similarity over the whole section of the motions of air in two smooth pipes of different diameters can only exist when the values of both μ' and μ are the same for each. From the value of μ' here determined this condition can only be satisfied if the values of $v_c l$ and μ are the same for the two pipes. To illustrate this similarity, experiments were made on the distribution of velocity in the two smooth pipes used for this work and in which the centre filament velocities were respectively 1525 cm. per second in the 4·93 cm. pipe and 1017 cm. per second in the 7·4 cm. pipe. The experimental results for both pipes are given in Table II and are also plotted in fig. V, and it will be seen that, within the limits of accuracy of the observations, the curves are identical from the centre to the walls. It is evident, therefore, that in all questions of the prediction of the frictional resistance of bodies moving in fluids from experiments on small models of them, so long as the form of the function $f(\nu/v_cl)$ remains unknown, the only method of comparison is that in



which the value of v_cl for the model is made the same as the corresponding quantity for the full-sized body.

The author desires to express his thanks to his colleague Mr. J. R. Pannell for the valuable assistance he has given in taking observations and reducing the results.